[SageMath](http://sagemath.org) en cinq minutes

Nicolas M. Thiéry (http://Nicolas.Thiery.name)
Laboratoire de Recherche en Informatique (http://lri.fr)
Université Paris Sud (http://www.u-psud.fr)

PyParis
Session Python pour l'Éducation
September 28rd of 2018

Ce document actif (https://mybinder.org/v2/gh/nthiery/shared-sage-notebooks/master?filepath=2018-11-14-PyParis-Education-SageMath.ipynb) sur Binder (https://mybinder.org)
Shorthand: https://tinyurl.com/pyparis18-education-sage

SageMath (sagemath.org) en deux mots

- Un système de calcul généraliste pour les mathématiques

- Basé sur Python (Python 2; port vers Python 3 en cours)
  Et une multitude de bibliothèques de calculs

- Licence libre, gratuit
- Multi-plateforme: Linux, Windows, MacOS
  ou en ligne (par exemple sur CoCalc (cocalc.com))

- Développé par une communauté d'enseignants, chercheurs, ingénieurs, ...
- 3-4 versions par an depuis 2005; ~100 contributeurs par version
- Financements: NSF (USA), H2020 (Europe), ...

- Utilisé à l'université (L1 -> agreg, doctorat), en recherche, ...
- Au lycée?

SageMath (http://sagemath.org) en démo

Arithmétique: de l'exact au numérique à précision arbitraire

In [1]:

```
1 + 1
```

Out[1]:

```
2
```

In [2]:

```
( 1 + 2 * (3 + 5)^2 ) * 2
```

Out[2]:

```
258
```

In [3]:

```
20/14
```

Out[3]:

```
10/7
```

In [4]:

```
2^1000
```

Out[4]:

```
1071508607186267320948425049060001810561404811705533607443750388370351051124936122493198378815695858127 59
```

In [5]:

```
20.0/14
```

Out[5]:

```
1.42857142857143
```

In [6]:

```
numerical_approx(4*arctan(1), 10000)
```

Out[6]:

```
3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798 21
```

Arithmétique: nombres premiers, modulo, corps fini; codage RSA?

In [7]:

```
p = next_prime(2^9);
p
```

Out[7]:

```
521
```

In [8]:

```
550 % p
```

Out[8]:

```
29
```

In [9]:

```
Zp = FiniteField(p)
```

In [10]:

```
a = Zp(550); a
```

Out[10]:

```
29
```

In [11]:

```
a^(7^7^7)
```

Out[11]:

```
226
```

In [12]:

```
Zp.category()
```

Out[12]:

```
Join of Category of finite enumerated fields and Category of subquotients of monoids and Category of quot
```

In [13]:

```
Zp.category().axioms()
```

Out[13]:

```
frozenset({'AdditiveAssociative',
           'AdditiveCommutative',
           'AdditiveInverse',
           'AdditiveUnital',
           'Associative',
           'Commutative',
           'Distributive',
           'Division',
           'Enumerated',
           'Finite',
           'NoZeroDivisors',
           'Unital'})
```

Un peu d'algèbre

In [14]:

```
var('x,y')
```

Out[14]:

(x, y)

In [15]:

```
factor(x^100 - 1)
```

Out[15]:

```
(x^40 - x^30 + x^20 - x^10 + 1)*(x^20 + x^15 + x^10 + x^5 + 1)*(x^20 - x^15 + x^10 - x^5 + 1)*(x^8 - x^6
```

In [16]:

```
%display latex
```

In [17]:

```
factor(x^100 -1)
```

Out[17]:

$$\left(x^{40}-x^{30}+x^{20}-x^{10}+1\right)\left(x^{20}+x^{15}+x^{10}+x^5+1\right)\left(x^{20}-x^{15}+x^{10}-x^5+1\right)\left(x^8-x^6+x^4-x^2+1\right)\left(x^4+x^3+x^2+x+1\right)$$

In [18]:

```
solve([x^2+y^2 == 1, y^2 == x^3 + x + 1], x, y)
```

Out[18]:

$$\left[\left[x=-\frac{1}{2}i\sqrt{3}-\frac{1}{2},y=-\sqrt{-\frac{1}{2}i\sqrt{3}+\frac{3}{2}}\right],\left[x=-\frac{1}{2}i\sqrt{3}-\frac{1}{2},y=\sqrt{-\frac{1}{2}i\sqrt{3}+\frac{3}{2}}\right],\left[x=\frac{1}{2}i\sqrt{3}-\frac{1}{2},y=-\sqrt{\frac{1}{2}i\sqrt{3}+\frac{3}{2}}\right],\left[x=\frac{1}{2}i\sqrt{3}-\frac{1}{2},y=\sqrt{\frac{1}{2}i\sqrt{3}+\frac{3}{2}}\right],[x=0,y=(-1)],[x=0,y=1]\right]$$

Un peu d'analyse

In [19]:

```
f = (cos(pi/4-x)-tan(x)) / (1-sin(pi/4 + x)); f
```

Out[19]:

$$-\frac{\cos\left(\frac{1}{4}\,\pi-x\right)-\tan(x)}{\sin\left(\frac{1}{4}\,\pi+x\right)-1}$$

In [20]:

```
f.derivative()
```

Out[20]:

$$\frac{\tan\left(x\right)^2-\sin\left(\frac{1}{4}\,\pi-x\right)+1}{\sin\left(\frac{1}{4}\,\pi+x\right)-1}+\frac{\left(\cos\left(\frac{1}{4}\,\pi-x\right)-\tan(x)\right)\cos\left(\frac{1}{4}\,\pi+x\right)}{\left(\sin\left(\frac{1}{4}\,\pi+x\right)-1\right)^2}$$

In [21]:

```
limit(f, x = pi/4, dir='minus')
```

Out[21]:

$+\infty$

In [22]:

```
f = cos(x)^6 + sin(x)^6 + 3 * sin(x)^2 * cos(x)^2; f
```

Out[22]:

$$\cos\left(x\right)^6+\sin\left(x\right)^6+3\,\cos\left(x\right)^2\sin\left(x\right)^2$$

In [23]:

```
f.simplify_trig()
```

Out[23]:

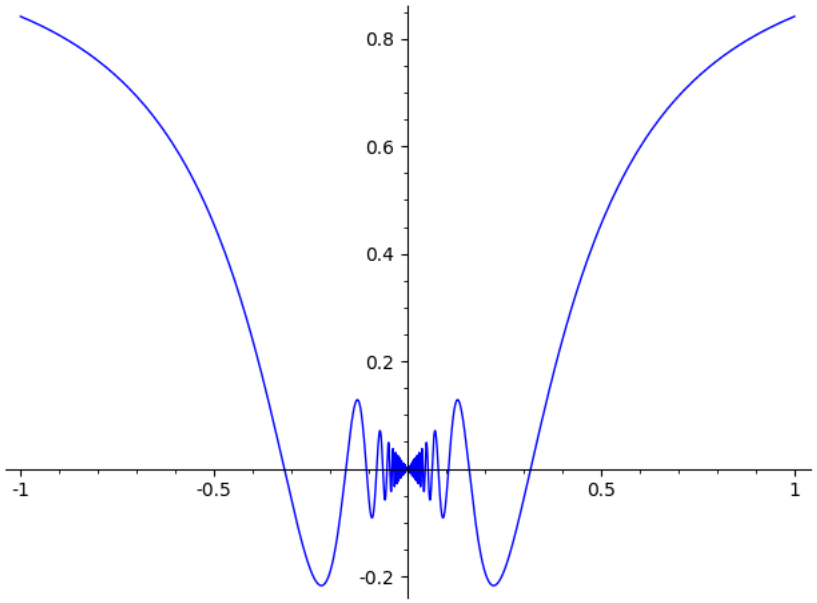
1

Graphiques

In [24]:

```
plot(x*sin(1/x))
```

Out[24]:



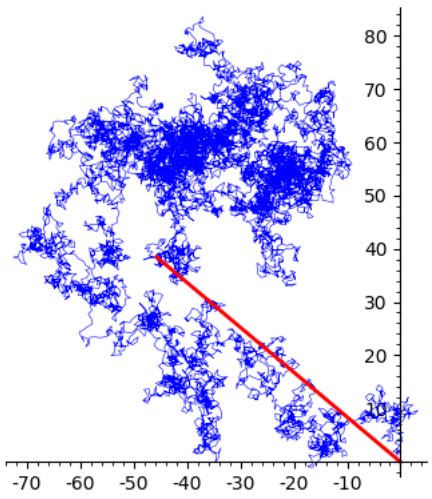
Graphique et programmation : marche aléatoire

In [25]:

```
n, l, x, y = 10000, 1, 0, 0
p = [[0, 0]]

for k in range(n):
    theta = (2 * pi * random()).n(digits=5)
    x, y = x + l * cos(theta), y + l * sin(theta)
    p.append([x, y])

g = line(p, thickness=.4) + line([p[n], [0, 0]], color='red', thickness=2)
g.show(aspect_ratio=1)
```



Graphiques interactifs

In [26]:

```
@interact
def g(f=x*sin(1/x),
      c=slider(-1, 1, .01, default=-.5),
      n=(1..30),
      xinterval=range_slider(-1, 1, .1, default=(-8,8), label="x-interval"),
      yinterval=range_slider(-1, 1, .1, default=(-3,3), label="y-interval")):
    x0 = c
    degree = n
    xmin,xmax = xinterval
    ymin,ymax = yinterval
    p   = plot(f, xmin, xmax, thickness=4)
    dot = point((x0,f(x=x0)),pointsize=80,rgbcolor=(1,0,0))
    ft = f.taylor(x,x0,degree)
    pt = plot(ft, xmin, xmax, color='red', thickness=2, fill=f)
    show(dot + p + pt, ymin=ymin, ymax=ymax, xmin=xmin, xmax=xmax)
    html('$f(x)\;=\;%s$'%latex(f))
    html('$P_{%s}(x)\;=\;%s+R_{%s}(x)$'%(degree,latex(ft),degree))
```

Graphiques 3D

In [27]:

```
var('x,y')
plot3d(sin(pi*sqrt(x^2+y^2)) / sqrt(x^2+y^2), (x,-5,5), (y,-5,5), viewer="threejs")
```

Out[27]:

In [28]:

```
polytopes.truncated_icosidodecahedron().plot(viewer="threejs")
```

Out[28]:

Combinatoire et probabilités discrètes

In [29]:

```
%display ascii_art
```

In [30]:

```
Symboles = Set(["Coeur", "Carreau", "Pique", "Trefle"])
Valeurs  = Set([2, 3, 4, 5, 6, 7, 8, 9, 10, "Valet", "Dame", "Roi", "As"])
Cartes   = cartesian_product([Valeurs, Symboles])
Mains    = Subsets(Cartes, 5)
```

In [31]:

```
Mains.random_element()
```

Out[31]:

```
{(8, 'Pique'), (6, 'Carreau'), (5, 'Trefle'), (6, 'Coeur'), ('Roi', 'Pique')}
```

In [32]:

```
Mains.cardinality()
```

Out[32]:

```
2598960
```

In [33]:

```
Couleurs = cartesian_product([Subsets(Valeurs, 5), Symboles])
Couleurs.random_element()
```

Out[33]:

```
( {'Roi', 'As', 4, 6, 'Dame'}, Coeur )
```

In [34]:

```
float( Couleurs.cardinality() / Mains.cardinality() )
```

Out[34]:

```
0.00198079231693
```

In [35]:

```
Mains.cardinality??
```

Théorie des graphes

In [36]:

```
graphs
```

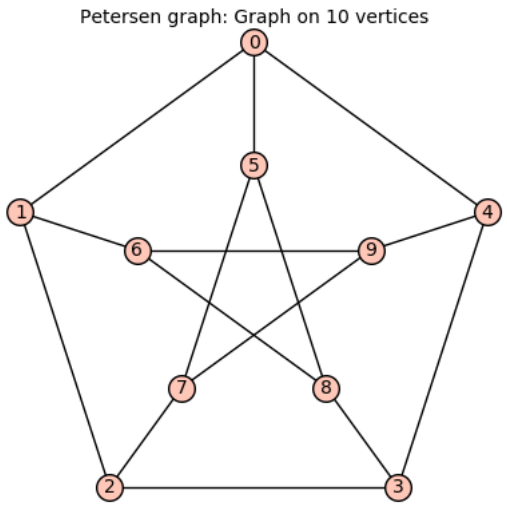
Out[36]:

```
<sage.graphs.graph_generators.GraphGenerators instance at 0x7f4a08cc7b00>
```

In [37]:

```
g = graphs.PetersenGraph(); g
```

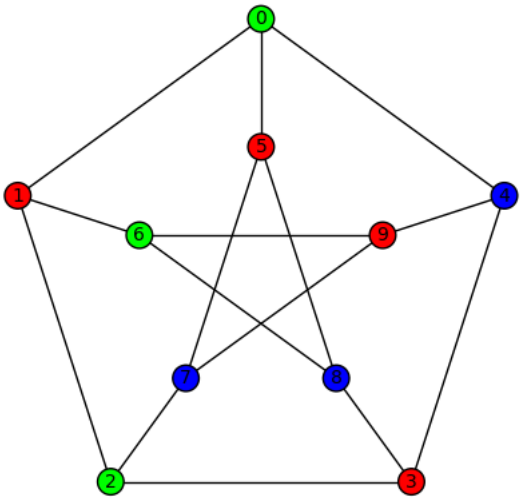
Out[37]:



In [38]:

```
g.plot(partition=g.coloring())
```

Out[38]:



SageMath

- Un environnement confortable et homogène (Jupyter)
- Pour la programmation et les mathématiques (Python)
- Libre, gratuit, pérenne, multiplateforme
- Inclut Numpy, Scipy, Matplotlib, ...

Livres

- Calcul Mathématique avec SageMath (http://sagebook.gforge.inria.fr/)
  2013, libre, 12€, en anglais (http://sagebook.gforge.inria.fr/english.html) (2018), en allemand (www.loria.fr/~zimmerma/sagebook/CalculDeutsch.pdf)
- SageMath for undergraduates (http://www.gregorybard.com/Sage.html), Gregory Bard
- SageMath au lycée???